# THE PROBLEM OF CALCULATING THE BOUNDARY LAYER ON THE INSULATION WALL OF AN MHD CHANNEL*

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We present the results of calculation on an M-20 computer of the laminar compressible boundary layer at the insulation wall in the channel of an MHD generator for the case of conductivity anisotropy and for a variable coefficient of electromagnetic load along the length of the channel.

The coefficient K of electromagnetic load is not generally constant in solutions of the equations for the boundary layer at the insulation wall in the channel of an MHD generator, but varies over the length of the channel as a result of changes in electric-field strength $E_y$ and in the flow velocity $u_\infty$ in the core.

The system of equations for the calculation of the boundary layer at the insulation wall [1], corresponding to a coefficient of electromagnetic load constant along the channel length, can be modified for the general case of a variable K by introduction of the following expressions:

$$\frac{K}{K-1}=\frac{K_*}{K_*-1}+a(8\xi), \qquad \text{(I)}$$

$$\frac{1}{K}=\frac{1}{K_*}+c(8\xi), \qquad \text{(II)}$$

$$\frac{1}{K-1}=\frac{1}{K_*-1}+d(8\xi), \qquad \text{(III)}$$

where $K_*$ is the coefficient of electromagnetic load at the initial cross section; $a$, c, and d are coefficients whose values are determined through solution of Eqs. (I)–(III). The function representing the change in K along the channel length and used in the solution of Eqs. (I)–(III) is found from the system of equations for the flow core.

With consideration of (I)–(III), assuming

$$\rho_\infty u_\infty^2\frac{du_\infty}{dx}=-j_{y\infty}E_y, \qquad (11_d)$$

$$j_{y\infty}=\sigma_\infty(u_\infty B_z-E_y), \qquad (23_d)$$

the system of equations subject to solution is written in the form:

1) for the equation of motion in the longitudinal direction

$$f_0'''+f_0f_0''=0, \qquad (24)$$

$$f_1'''+f_0f_1''-2f_0'f_1'+3f_0''f_1=-\theta_0+$$
$$+\frac{\theta_0}{K_*}\left\{1-\Pi\left[\left(\frac{K_*}{K_*-1}\right)+\right.\right. \qquad (25)$$

*Addition to article [1].

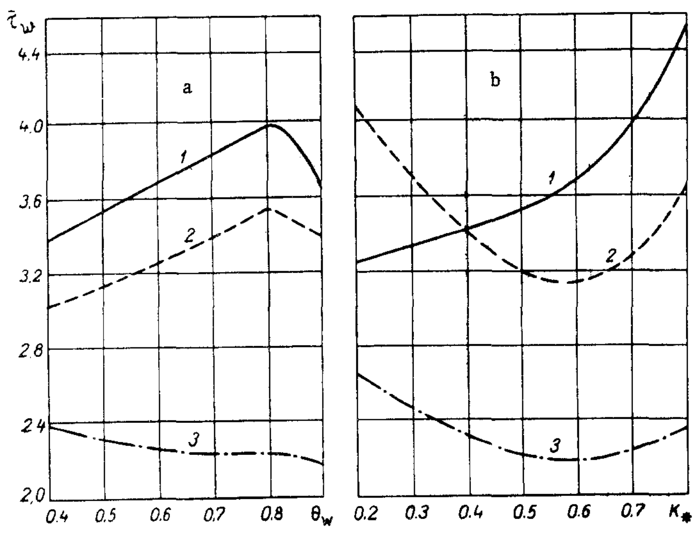

Fig. 1. Change in friction coefficient $\bar{\tau}_w$ as function of Hall parameter at various wall temperatures and load coefficients; (1) $\beta e\infty = 0$; 2) 1.0; 3) 2.5): a) $\bar{\tau}_w = F_1(\theta_w)$; $\xi = 0.05$; $K_* = 0.5$; b) $\bar{\tau}_w = F_1(K_*)$; $\xi = 0.05$; $\theta_w = 0.6$.

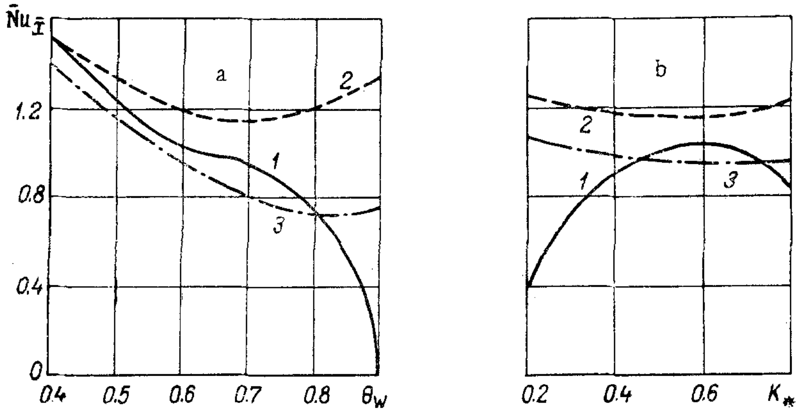


Fig. 2. Change in coefficient $\overline{Nu}_{\bar{x}}$ as a function of Hall parameter (See Fig. 1); a) $\overline{Nu}_{\bar{x}} = F_2(\theta_w)$; $\xi = 0.05$; $K_* = 0.5$; b) $\overline{Nu}_{\bar{x}} = F_2(K_*)$; $\xi = 0.05$; $\theta_w = 0.6$.

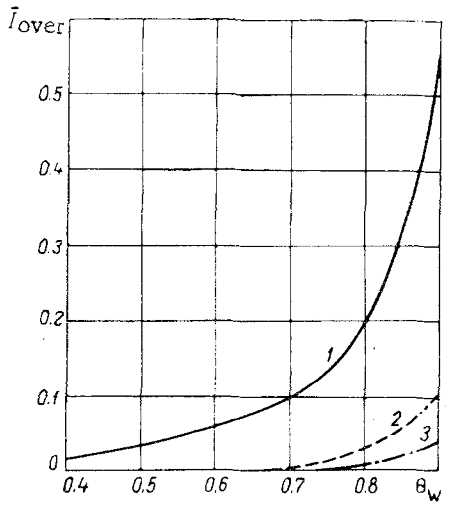


Fig. 3. Change in current of outflow $\bar{I}_{over}$ as a function of wall temperature for conditions $\xi = 0.05$; $K_* = 0.5$ (See Fig. 1).

$$+\beta_{e\infty}^2\theta_0^{\frac{1}{2}}+\left(\frac{1}{K_*-1}\right)\frac{f_0'}{2}\Big]\Big\}, \quad (25) \text{ (cont'd)}$$

$$f_2'''+f_0f_2''-4f_0'f_2'+5f_0''f_2=$$
$$=2f_1'^2-3f_1f_1''-\left(\theta_1+\frac{\theta_0}{8}\right)\left(1-\frac{1}{K_*}\right)-$$
$$-\frac{\Pi}{K_*}\Big\{\Big[\left(\frac{K_*}{K_*-1}\right)+\beta_{e\infty}^2\theta_0^{\frac{1}{2}}\Big]\left(\theta_1+\frac{\theta_0}{8}\right)-$$
$$-\left(\frac{1}{K_*-1}\right)\Big[\frac{f_0'}{2}\left(\theta_1+\frac{\theta_0}{8}\right)+$$
$$+\theta_0\left(\frac{f_1'}{2}-\frac{f_0'}{16}-\beta_{e\infty}\theta_0^{\frac{1}{2}}r_1\right)\Big]\Big\}+A(\eta), \quad (26_d)$$

2) for the equation of motion in the transverse direction

$$r_1''+f_0r_1'-2r_1f_0'=$$
$$=\frac{\Pi}{2K_*}\Big\{\Big[\beta_{e\infty}-\left(\frac{K_*}{K_*-1}\right)\beta_{e\infty}\theta_0^{\frac{1}{2}}\Big]\theta_0+$$
$$+\left(\frac{1}{K_*-1}\right)\frac{\beta_{e\infty}}{2}f_0'\theta_0^{3/2}\Big\}, \quad (27)$$

$$r_2''+f_0r_2'-4f_0'r_2=2r_1f_1'-3f_1r_1'+\frac{\Pi}{2K_*}\Big\{\Big[\beta_{e\infty}-$$
$$-\left(\frac{K_*}{K_*-1}\right)\beta_{e\infty}\theta_0^{\frac{1}{2}}\Big]\left(\theta_1+\frac{\theta_0}{8}\right)+\left(\frac{1}{K_*-1}\right)\times$$
$$\times\Big[\frac{\beta_{e\infty}}{2}f_0'\theta_0^{1/2}\left(\theta_1+\frac{\theta_0}{8}\right)\Big]\Big\}-\frac{\Pi}{2K_*}\left(\frac{1}{K_*-1}\right)\times$$
$$\times\Big[-\beta_{e\infty}\theta_0^{1/2}\left(\frac{f_1'}{2}-\frac{f_0'}{16}\right)-r_1\Big]\theta_0+B(\eta), \quad (28_d)$$

3) for the equation of energy

$$\frac{1}{\Pr_*}\theta_0''+f_0\theta_0'=-\beta_1f_0''^2, \quad (29)$$

$$\frac{1}{\Pr_*}\theta_1''+f_0\theta_1'-2f_0'\theta_1=$$
$$=-3\theta_0'f_1-\beta_1\left[f_0f_0'f_1''+3f_0'f_0''f_1+f_0f_1'f_0''-\right.$$
$$\left.-2f_0''^2f_1'+2f_0''f_1''+f_1'f_1''+f_1''f_0'\right]+$$
$$+2\beta_1\Gamma\Big\{\Big[\beta_{e\infty}-\left(\frac{K_*}{K_*-1}\right)\beta_{e\infty}\theta_0^{1/2}\Big]\theta_0+$$
$$+\frac{1}{(K_*-1)}\frac{\beta_{e\infty}}{2}f_0'\theta_0^{3/2}\Big\}-2\beta_1\Pi\Big\{\Big[\left(\frac{K_*}{K_*-1}\right)+$$
$$+\beta_{e\infty}^2\theta_0^{1/2}\Big]\theta_0-\left(\frac{1}{K_*-1}\right)\frac{f_0'}{2}\theta_0\Big\}, \quad (30)$$

$$\frac{1}{\Pr_*}\theta_2''+f_0\theta_2'-4f_0'\theta_2=$$
$$=2\theta_1f_1'-3\theta_1'f_1-5\theta_0'f_2+\beta_1\left[4f_0''^2f_2'+4f_0'f_1''^2-\right.$$
$$-5f_0'f_0''f_2-3f_0'f_1f_1''-f_0f_0'f_2''-f_0f_1'f_1''-3f_0''f_1f_1'-$$
$$\left.-f_0f_0''f_2'-f_2''f_0'-f_1'f_1''-f_1''^2-2f_0''f_2''-f_0'''f_2'\right]+$$
$$+4\beta_1\left[2r_1^2f_0'-r_1r_1'f_0-r_1'^2-r_1r_1''\right]+$$
$$+2\beta_1\Gamma\Big\{\Big[\beta_{e\infty}-\left(\frac{K_*}{K_*-1}\right)\beta_{e\infty}\theta_0^{\frac{1}{2}}\Big]\left(\theta_1+\frac{\theta_0}{4}\right)+$$
$$+\left(\frac{1}{K_*-1}\right)\frac{\beta_{e\infty}}{2}f_0'\theta_0^{\frac{1}{2}}\left(\theta_1+\frac{\theta_0}{4}\right)\Big\}-$$
$$-2\beta_1\Gamma\left(\frac{1}{K_*-1}\right)\times$$
$$\times\Big[-\beta_{e\infty}\theta_0^{1/2}\frac{f_1'}{2}+\frac{1}{16}f_0'\beta_{e\infty}\theta_0^{1/2}-r_1\Big]\theta_0-$$
$$-2\beta_1\Pi\Big\{\Big[\left(\frac{K_*}{K_*-1}\right)+\beta_{e\infty}^2\theta_0^{1/2}-$$
$$-\left(\frac{1}{K_*-1}\right)\frac{f_0'}{2}\Big]\left(\theta_1+\frac{\theta_0}{4}\right)\Big\}-2\beta_1\Pi\left(\frac{1}{K_*-1}\right)\times$$
$$\times\left(-\frac{f_1'}{2}+\frac{f_0'}{16}+\beta_{e\infty}\theta_0^{1/2}r_1\right)\theta_0+C(\eta), \quad (31_d)$$

In these equations we have denoted:

$$A(\eta)=\frac{\theta_0}{K_*}\Pi\left(a+d\frac{f_0'}{2}\right)-$$
$$-\theta_0C\Big\{1-\Pi\Big[\left(\frac{K_*}{K_*-1}\right)+$$
$$+\beta_{e\infty}^2\theta_0^{\frac{1}{2}}+\left(\frac{1}{K_*-1}\right)\frac{f_0'}{2}\Big]\Big\},$$

$$B(\eta)=\frac{\Pi}{2K_*}\left(a-d\frac{f_0'}{2}\right)\beta_{e\infty}\theta_0^{3/2}-\frac{\Pi}{2}C\Big[1-$$
$$-\left(\frac{K_*}{K_*-1}\right)\theta_0^{\frac{1}{2}}+\left(\frac{1}{K_*-1}\right)\frac{f_0'}{2}\theta_0^{\frac{1}{2}}\Big]\beta_{e\infty}\theta_0,$$

$$C(\eta)=2\beta_1\Gamma\left(a-d\frac{f_0'}{2}\right)\beta_{e\infty}\theta_0^{3/2}+$$
$$+2\beta_1\Gamma\Big\{\beta_{e\infty}-\left(\frac{K_*}{K_*-1}\right)\beta_{e\infty}\theta_0^{\frac{1}{2}}+$$
$$+\left(\frac{1}{K_*-1}\right)\beta_{e\infty}\frac{f_0'}{2}\theta_0^{\frac{1}{2}}\Big\}\theta_0+2\beta_1\Pi\left(a-d\frac{f_0'}{2}\right)\theta_0.$$

We are not repeating the equations of [1] here, since these are retained without change. The equations differing from those in [1] are indicated by the subscript "d."

The changes in the notation of the equations relative to those appearing in [1] are a result of the introduction of the variable K, as well as of the typographical errors occurring in [1] in connection with the mathematical signs appearing before the current terms in Eqs. (11) and (23). The notation of Eqs. (11) and (23)–(31), presented in [1], with consideration of these typographical errors, must correspond to Eqs. ($11_d$) and ($23_d$)–($31_d$) of this addition.

We can see that additional terms which take into consideration the change in K along the length of the channel appear in the equations for the second approximation ($26_d$), ($28_d$), and ($31_d$).

The system of equations ($24_d$)–($31_d$) was solved on an M-20 computer (the programming and calculation of the problem were carried out at the computer research department of the Mozhaiskii LVIKA by T. S. Kuznichenkova) for a change in wall temperature within the range $\theta_w = 0.4–0.9$, for a change in the electromagnetic-load coefficient within the limits $K_* = 0.2 - 0.8$, and for a change in the Hall parameter $\beta_{e\infty} = 0 \div 5$.

The equations were solved by well-known methods for the following conditions: plasma—helium + 1.5% potassium by weight; $\gamma = 1.67$; $\chi = 7.5$; $\Pr_* = 0.7$; $m = 0.1$; $P_* = 12 \cdot 10^4\,\mathrm{N/m^2}$; $M_* = 0.9$; $T_* = 2400°\,\mathrm{K}$; $B_z = 2\,\mathrm{T}$; $b = 2.2 \cdot 10^{-2}\,\mathrm{m}$; $a = 0.42$; $\beta_1 = 0.164$; $d = 0.4$; $c = 0.62$.

Figures 1–3 show changes in the frictional stresses $\bar{\tau}_w$, in the Nusselt number $\overline{\mathrm{Nu}}_{\bar{x}}$, and in the total overflow current $\bar{I}_{\mathrm{over}}$ as a function of the wall temperature and of the loading factor at the inlet for three values of the Hall parameter. The quantities $\bar{\tau}_w$, $\overline{\mathrm{Nu}}_{\bar{x}}$, and $\bar{I}_{\mathrm{over}}$ are calculated from the equations

$$\bar{\tau}_w = \frac{\tau_w}{\mu_* \dfrac{u_*}{4}\left(\dfrac{u_* L}{\nu_*}\right)^{1/2}} = \bar{P}\left\{\left[\frac{S}{\bar{P}^2 m}\left(\frac{K}{K-1}\right)\frac{\bar{j}_{y\infty}^2}{\bar{\sigma}_\infty} + 1\right] \times \frac{1}{2m}\right\}^{-\frac{1}{2}} \left[f_0''(0) - (8\xi) f_1''(0) + (8\xi)^2 f_2''(0) - \ldots\right],$$

$$\overline{\mathrm{Nu}}_{\bar{x}} = \frac{\mathrm{Nu}_{\bar{x}}}{\left(\dfrac{u_* L}{4\nu_*}\right)^{\frac{1}{2}}} = \bar{P}\left\{\left[\frac{S}{\bar{P}^2 m} \times \left(\frac{K}{K-1}\right)\frac{\bar{j}_{y\infty}^2}{\bar{\sigma}_\infty} + 1\right]\frac{1}{2m}\right\}^{-\frac{1}{2}} \times \frac{1}{\theta_w(1-\theta_w)}\{\theta_0(0) - (8\xi)\theta_1(0) + (8\xi)^2\theta_2(0) - \ldots\},$$

$$\bar{I}_{\mathrm{over}} = \frac{\int_0^x\int_0^{\delta_1}(j_y - j_{y\infty})\,dz\,dx + \int_0^x\int_{b-\delta_2}^{b}(j_y - j_{y\infty})\,dz\,dx}{\int_0^x\int_0^b j_y\,dz\,dx},$$

where S is the parameter of electromagnetic interaction; b is the channel height; $\bar{\sigma}_\infty = \sigma_\infty/\sigma_*$; $\bar{j}_{y\infty} = j_{y\infty}/\sigma_* u_* B_{z*}$; $\bar{P} = P/P_*$.

We should note that in the equation for $\bar{\tau}_w$, the contribution from the gradient of the transverse velocity component is neglected because of its smallness.

The examination of the calculation variants makes it possible to draw conclusions as to the relatively small values for the overflow currents over the entire subject range of wall temperatures $\theta_w = 0.4–0.9$, even for small dimensions of the MHD channel, for noticeable changes in the frictional stress and in the Nusselt number with an increase in the Hall parameter, as well as making it possible to draw conclusions regarding the substantial effect exerted on $\bar{\tau}_w$ and $\overline{\mathrm{Nu}}_{\bar{x}}$ by the coefficient of electromagnetic loading.

## REFERENCES

1. S. M. Apollonskii, IFZh [Journal of Engineering Physics], 10, no. 4, 1966.

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